Q = gas flow rate at 25°C and 1 atm, cm^3/s
R = radius of spherical particle, cm
t = time, s
V_B = bubble volume per unit volume of bubble and solid free liquid
V_L = total volume of liquid in the vessel, cm^3
v_B = vertical bubble velocity, cm/s

Greek Letters

α = bubble-to-liquid rate parameter defined by Equation (3), cm^{-1}
β = particle porosity
γ = tortuosity factor
$\mu'_{1,D}$ or $\mu'_{1,d.v.}$ = first absolute moment measured at the detector, for slurry system or for dead volume, respectively, s
$\mu_{2,D}$ or $\mu_{2,d.v.}$ = second central moment measured at the detector for slurry system or for dead volume, respectively, s^2
ρ_p = particle density, g/cm^3

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Analysis of Two-Dimensional Air-Bubble Plumes

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In recent years, air-bubble systems have been applied successfully for many purposes, such as prevention of ice formation in lakes, as barriers against saltwater intrusion in rivers and lakes, for stopping the spreading of oil spills on water surface, for reduction of underwater explosion waves, and for aeration in water purification and waste treatment plants.

Two dimensional air-bubble plumes will exist when air is released from either a perforated tube or a tube with a longitudinal slot submerged in water. No suitable analytical model has been developed so far to describe the two-dimensional, two-phase plume when the density difference of the fluid inside the jet and that of the surrounding fluid, such as in the air-liquid case, is very large.

The purpose of this study is to extend the two-phase axisymmetric jet model, developed by Hussain and Siegel (1976), to the two-dimensional case. The proposed entrainment process, to account for the contributions to the entrainment by the outer liquid flow, the bubble wakes, and the rising air bubbles, is similar to that of Hussain and Siegel (1976).

ANALYSIS

The two-dimensional, two-phase jet considered here is similar to that of Figure 1 of Hussain and Siegel except that the circular orifice is replaced by a slot or a row of orifices. The flow field is assumed to be steady, isothermal, and fully turbulent. The density of the liquid is assumed constant, while the gas density is assumed to vary according to the ideal gas law. As in the work of Hussain and Siegel (1976), no attempt has been made to analyze the jet's turning zone when it reaches the liquid surface. It is also assumed that the gas leaves the submerged pipe with a negligible upward momentum. The bubbles are assumed to be sufficiently large so that their drag is fully turbulent, and hence they rise at constant terminal velocity relative to the liquid. The local bubble velocity is assumed equal to the local liquid velocity plus the bubble terminal velocity.

Gas Continuity

It is assumed that in the central region of the jet, bubbles are rising in a chain bubble fashion and that $1/(K+1)$ of the vertical height is occupied by the gas bubbles and $K/(K+1)$ is occupied by the liquid wake. This is shown in Figure 1. For a fixed gas flow rate, the mass flow rate M_g of the gas at any depth x can be written as

$$M_g = \int_o^{a_c} \frac{2}{K+1} U_g \rho_g dy = \text{constant} \qquad (1)$$

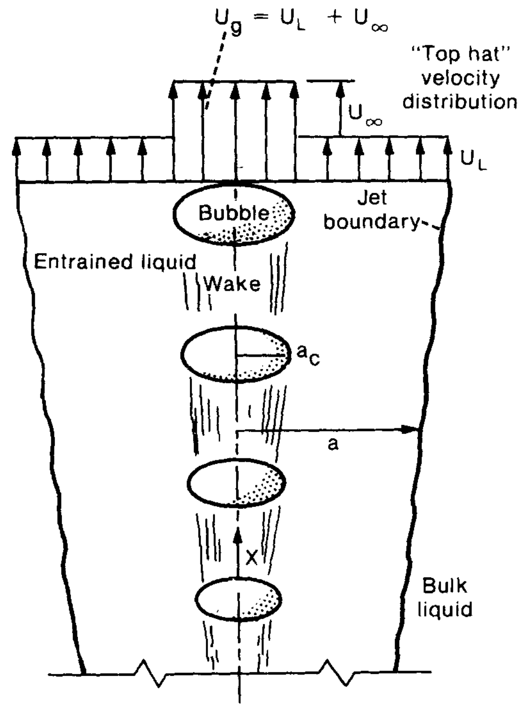

Fig. 1. Details of the proposed mathematical model for studying the two-phase jet.

where the gas density ρ_g is given by the ideal gas law

$$\rho_g = \frac{1}{RT}\left[P_a + \rho_l(L - x)\right] \quad (2)$$

Liquid Continuity

The following equation accounts for the liquid carried into the liquid region and also into the bubble wakes of the jet by turbulent entrainment:

$$\frac{d}{dx}\left[\int_o^{a_c} \frac{K}{K+1}\rho_l U_g dy + \int_{a_c}^{\infty} \rho_l U_l dy\right] = -\left(\rho_l V\right)_{y\to\infty} = E\rho_l U_l \quad (3)$$

Gas and Liquid Momentum

The upward buoyancy force of the bubbles is balanced by a small change of bubble momentum and a large drag force F_D of the liquid on the bubbles

$$\int_o^{a_c} \frac{g}{K+1}(\rho_l - \rho_g)dy = \frac{d}{dx}\left[\int_o^{a_c} \frac{1}{K+1}\rho_g U_g^2 dy\right] + F_D \quad (4)$$

where

$$U_l(x, y) = \begin{cases} U_l(x) & a_c(x) \leq y \leq a(x) \\ o & a(x) < y \end{cases} \quad (5b)$$

$$U_g(x) = U_l(x) + U_\infty \quad (6)$$

Expression for Entrainment Coefficient E

The term $E\rho_l U_l$ in Equation (3) represents the total mass being physically entrained at the jet's boundary. For a single-phase jet, when the density in the plume is the same as that of the surrounding fluid, E is taken as a constant. Ricou and Spalding (1961) suggest that for the case of a single-phase axisymmetric jet, the value of E should be modified by a multiplicative factor of $(\rho_l/\rho_o)^{1/2}$. Based on the concept that the entrainment depends on the square root of the excess *momentum flux* in the jet, Hussain and Siegel further suggest that the entrainment also depends on the velocity of the jet relative to its surroundings and on the interfacial area between the jet and the surrounding region. Accordingly, the expression for entrainment can be written as

$$E = E_o\left[1 + \frac{K}{K+1}\frac{U_\infty}{U_l} + \frac{1}{K+1}\left(\frac{\rho_g}{\rho_l}\right)^{1/2}\frac{U_\infty}{U_l}\right] \quad (7)$$

Equations (2), (5), (6), and (7) together with Equations (1), (3), and (4) result in the following:

$$M_g = \frac{2}{K+1}a_c U_g \rho_g = \text{constant} \quad (8)$$

$$\frac{-K}{2}\frac{M_g}{\rho_g^2}\frac{d\rho_g}{dx} + U_l\left(\frac{da}{dx} - \frac{da_c}{dx}\right) + (a - a_c)\frac{dU_l}{dx} = E_o U_l\left[1 + \frac{K}{K+1}\frac{U_\infty}{U_l} + \frac{1}{K+1}\frac{U_\infty}{U_l}\left(\frac{\rho_g}{\rho_l}\right)^{1/2}\right] \quad (9)$$

$$\frac{K}{2}M_g\frac{d}{dx}\left(\frac{U_g}{\rho_g}\right) + \frac{M_g}{2\rho_l}\frac{dU_g}{dx} + \frac{d}{dx}\left[U_l^2\left(a - \frac{K+1}{2}\frac{M_g}{\rho_g U_g}\right)\right] = \frac{gM_g}{2U_g\rho_g}\left(1 - \frac{\rho_g}{\rho_l}\right) \quad (10)$$

Also, the mass flow rate of the liquid M_l pumped by the rising gas is

$$M_l = 2(a - a_c)\rho_l U_l + KM_g\frac{\rho_l}{\rho_g} \quad (11)$$

Equations (8), (9), and (10) can be rearranged to yield the following two simultaneous differential equations:

$$\frac{du_l}{dx} = \frac{2E_oU_\infty U\left[\frac{\rho_g U_l}{M_g U_\infty} + \frac{\rho_g}{(K+1)M_g}\left\{K + \left(\frac{\rho_g}{\rho_l}\right)^{1/2}\right\}\right] - \frac{g\left(1 - \frac{\rho_g}{\rho_l}\right)}{(U_l + U_\infty)} + \frac{KU_\infty}{RT}\frac{\rho_l}{\rho_g}}{\frac{(K+1)U_l}{U_l + U_\infty} - \frac{2aU_l\rho_g}{M_g} - \left(K + \frac{\rho_g}{\rho_l}\right)} = F(U_l, a, x) \quad (12)$$

$$F_D = \frac{d}{dx}\left[\int_o^{a_c} \frac{K}{K+1}\rho_l U_g^2 dy + \int_{a_c}^{\infty} \rho_l U_l^2 dy\right]$$

Top hat velocity profiles are used for evaluating the integrals of Equations (1), (3), and (4); namely

$$U_g(x, y) = U_g(x) \quad o \leq y \leq a_c(x) \quad (5a)$$

$$\frac{da}{dx} = E_o\left[1 + \frac{K}{K+1}\frac{U_\infty}{U_l} + \frac{1}{K+1}\frac{U_\infty}{U_l}\left(\frac{\rho_g}{\rho_l}\right)^{1/2}\right] + \frac{M_g\left(1 - K\frac{U_\infty}{U_l}\right)}{2\rho_g^2(U_l + U_\infty)}\frac{\rho_l}{RT} + \left\{\frac{(K+1)M_g\frac{U_\infty}{U_l}}{2\rho_g(U_l + U_\infty)^2}\right.$$

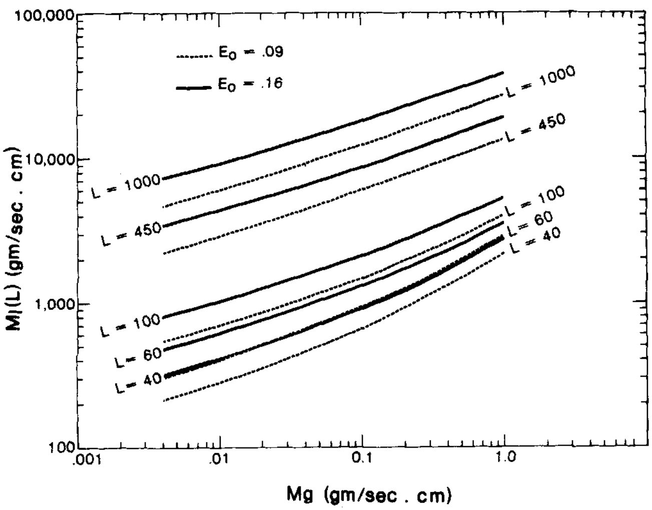

Fig. 2. Liquid pumped to the surface for various depths vs. gas flow rate.

$$\left. - \frac{a}{U_l} \right] F(U_l,a,x) \quad (13)$$

Numerical Solution

Standard IBM Library subroutine on Runge-Kutta method was used to solve Equations (12) and (13) simultaneously for U_l and a, respectively. At the starting boundary, that is, $x = o$, it was assumed that $U_l = o$, $U_g = U_\infty$ and $a = a_c$. The starting value of a is given by Equation (8) as

$$a(x = o) = a_c = M_g(K + 1) \Big/ \left(2U_\infty \frac{P_a + \rho_l L}{RT} \right) \quad (14)$$

Owing to the singularity in Equation (13) at $U_l = o$, at the starting boundary, U_l was assumed to be equal to 0.01. Calculations using smaller starting values of U_l showed no significant change.

RESULTS AND DISCUSSION

The results that follow are with regard to the prevention of ice formation in lakes by raising warm bottom water to the surface. The bulk water temperature is assumed to be 275°K. The terminal velocity U_∞ of a single bubble in undisturbed fluid was obtained from Zuber and Findley (1965) to be

$$U_\infty = 1.53 \left[\sigma g \frac{(\rho_l - \rho_g)}{\rho_l^2} \right]^{1/4} \quad (15)$$

A significant source of uncertainty is in specifying the entrainment coefficient E_o. Morton et al. (1956) suggested using $E_o = 0.093$, while a value of 0.16 was used by Lee and Emmons (1961) for a single-phase, two-dimensional plume. In the absence of any better information for E_o, for a two-phase jet such as for the present case, it was decided to obtain two sets of calculations using $E_o = 0.09$ and 0.16.

Figure 2 shows that the liquid pumped to the surface increases with the increasing depth of gas source. For depths greater than 4.5 m, M_l is linearly proportional to $M_g^{0.31}$. A cross plot for a given M_g, for $L > 4.5$ m, shows that $M_l \propto L^{1.125}$. Therefore, an approximate correlation of the calculated results for $L > 4.5$ m is given by

$$M_l = C M_g^{0.31} L^{1.125} \quad (16)$$

TABLE 1. VOLUMETRIC PUMPING RATES IN 2-D JET AT A LOCATION 3.3 M ABOVE A ROW OF ORIFICES SUBMERGED 4.3 M BELOW SURFACE

Volumetric air flow rates at standard conditions, Q_g cm³/s-m	$\frac{\text{Volume water pumped}}{\text{Volume air released}} = \frac{Q_l}{Q_g}$			
	Kobus (1968)	Balanin et al. (1970)	Present calculations $E_o = 0.09$	Present calculations $E_o = 0.16$
3 000	105	142.4	118.9	176.4
6 200	75	87.9	72.8	107.3
10 000	61	63.9	52.9	77.7

TABLE 2. LIQUID PUMPING RATES IN 2-D JETS FOR VARIOUS SUBMERGENCE DEPTHS OF A ROW OF ORIFICES AND VARYING GAS MASS FLOW RATES

		$M_l(L)$, g/s-cm		
L, cm	M_g, g/s-cm	Balanin et al. (1970)	Present study $E_o = 0.09$	Present study $E_o = 0.16$
100	0.0037	0.063	0.052	0.080
	0.0107	0.090	0.070	0.106
	0.1000	0.189	0.1460	0.212
	1.0	0.408	0.387	0.522
400	0.0037	0.210	0.199	0.309
	0.0107	0.298	0.265	0.401
	0.10	0.628	0.528	0.780
	1.0	1.353	1.172	1.690
800	0.0037	0.436	0.380	0.592
	0.0107	0.620	0.503	0.764
	0.10	1.361	0.991	1.468
	1.0	2.813	2.13	3.111
1 000	0.0037	0.553	0.466	0.728
	0.0107	0.787	0.614	0.935
	0.1	1.658	1.208	1.791
	1.0	3.572	2.580	3.776

where $C = 11.5$ for $E_o = 0.09$ and $C = 16.5$ for $E_o = 0.16$ when M_l and M_g are measured in grams per second-centimeters and L is in centimeters. For depths less than 4.5 m, the results are not linear on the log-log graph, and no attempt has been made to obtain an approximate correlation.

Table 1 shows comparison of the present study with the limited amount of available data. The calculated results obtained from the semiempirical correlation suggested by Balanin et al. (1970)

$$Q_l = 0.75 \left[(10 + L) L^2 \ln\left(1 + \frac{L}{10} \right) \frac{L}{10} \right]^{1/3} Q_g^{1/3} \quad (17)$$

for depths greater than 1 m are also given in Tables 1 and 2. The dimensions of Q_l and Q_g in the above relation are in cubic meters per second-meter, and L is in meters. The present study gives higher results than those of Kobus (1968) and Balanin (1970) for $E_o = 0.16$, while for $E_o = 0.09$, the results are generally lower. The disagreement between the present study and the other results is not serious, and it is encouraging to note that the proposed model here accurately predicts the reported trend; that is, that the pumped mass of water per unit mass of air decreases as the air mass flow rate increases, so that a more efficient pumping system is obtained at low flow rates. This is clearly shown in Figure 2. Figure 3 shows the calculated half width of the

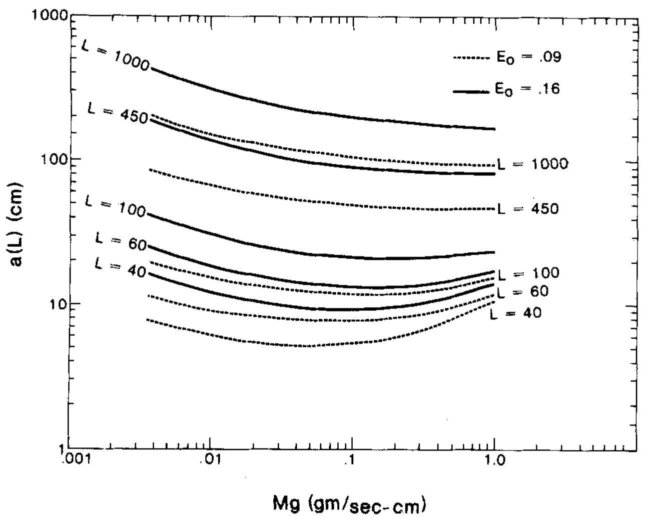

Fig. 3. Half width of jet at liquid surface vs. mass flow rate of air.

jet at the surface for a given value of M_g and L. The minor disagreement between the present results and the limited data available is due to the uncertainty in specifying accurately the value of E_o. More experimental work is needed to determine the correct value of E_o. However, it is felt that the present model is well suited for predicting accurately the liquid pumping capability of the two-phase, two-dimensional jet.

NOTATION

a = half width of jet, cm
K = ratio of wake volume to bubble volume = 1.5, see Hussain and Siegal (1976)
E = entrainment coefficient
E_o = entrainment constant
L = depth of row of orifices below liquid surface, cm
M = mass flow rate per cm of slot length, g/s-cm
P = pressure, g/cm^2
Q = volume flow rate per meter of slot length, cm^3/s-m
R = gas constant, g cm/g °K
T = absolute temperature, °K
U = fluid velocity, cm/s
ρ = fluid density, g/cm^3
σ = surface tension

Subscripts

a = atmosphere
c = jet's central bubbly region
g = gas
i = inside the jet, $y < a$
l = liquid
∞ = terminal
o = outside the jet, $y > a$

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On Desorption of Air from Liquid in Gas Absorption

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In the experiment (Mitsutake, 1973) of the the absorption of carbon dioxide into liquids, even in the case of small content of air in the liquids, air was liberated from the liquids. No experiments have yet been made about the effect of the desorption of air from liquids on the absorption of gas into them. The analogous experiments reported so far follow.

Dwyer and Dodge (1941) reported that the effect of humidity of the inlet gas on the rate of absorption of ammonia from air by water in a packed tower was quite small and within the experimental error. Grenier (1966) studied the effect of the counterdiffusion of water vapor on ammonia-air-hydrogen mixture on the absorption of ammonia into a water jet. Bourne (1969) and Tanaka et al. (1976) studied the effect of heat liberated by the dissolving ammonia and that of the temperature of the interface (by the evaporation of water) on the absorption of ammonia into water.

Again, as for our experiments, when pure carbon dioxide gas is absorbed in a bubble column, air liberated